# A multiscale description of failure in granular materials

Nejib Hadda, François Nicot, Luc Sibille, Farhang Radjai, Antoinette Tordesillas et al.

Citation: AIP Conf. Proc. **1542**, 585 (2013); doi: 10.1063/1.4811999
View online: http://dx.doi.org/10.1063/1.4811999
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1542&Issue=1
Published by the AIP Publishing LLC.

# A Multiscale Description of Failure in Granular Materials

Nejib Hadda[1], François Nicot[1], Luc Sibille[2], Farhang Radjai[3], Antoinette Tordesillas[4], and Félix Darve[5]

[1] *IRSTEA, Center of Grenoble, France*
[2] *LUNAM university, GeM institute, CNRS, Nantes, France*
[3] *LMGC laboratory, University of Montpellier, CNRS, France*
[4] *Department of Mathematics and Statistics, University of Melbourne, Victoria 3010, Australia*
[5] *UJF-INPG-CNRS, 3S-R Laboratory, Grenoble, France*

**Abstract.** This paper presents conditions of initiation and development of failure in granular materials through a two-dimensional discrete element model. General condition for the effective development of failure and its physical characteristics are recalled. Then relation between failure and the second order work expressed in terms of microscopic variables is discussed. Eventually, correspondence between a localized mode of failure marked with shear band patterns and space distribution of negative values of microscopic second-order work is investigated.

**Keywords:** failure, strain localization, second-order work, micro-mechanics, discrete element method.
**PACS:** 62.20.-x; 62.20.M-; 62.40.+i

## INTRODUCTION

Failure in granular materials is classically associated to the notion of plastic limit condition represented in the stress space by a Mohr-Coulomb-type failure surface. The objective of this paper is to reconsider the question of failure from a more general point of view. Hence we recall first: what is the physical manifestation of failure, the relation between its effective occurrence and the mode of control of the mechanical state, and finally the way to detect with the second-order work criterion the possible occurrence of failure. Then, in the framework of this general description of failure, we focus on the description of the mode of failure, either localized or diffused according to the development or the lack of localization of deformations. In particular, we investigate if a local expression of the second-order work (built from microscopic variables), in direct relation with the macroscopic expression of the second order work and thus with failure at the scale of the representative elementary volume, could also be linked to the mode of failure.

Discussions and analyses are based on numerical simulations performed with the discrete element based software YADE [1]. The granular assemblies are made of elastic circular particles (in 2D), or spherical ones (in 3D), and inter-particle contacts are purely frictional. Besides, we will make reference to the macroscopic expression of the second order work $W_2$, basically defined for a homogeneous volume $V_o$ in equilibrium at a given time $t$ under a prescribed external loading as:

$$W_2 = \int_{V_o} \delta\Pi_{ij} \frac{\partial(\delta u_i)}{\partial X_j} dV_o \qquad (1)$$

involving both incremental Piola-Kirchoff stress $\overline{\overline{\Pi}}$ and strain $\overline{\overline{F}}$, with $F_{ij} = \frac{\partial(\delta u_i)}{\partial X_j}$ experienced by the system during a time increment $\delta t$. In what follows, the lagrangian stress will be confounded with the usual Cauchy stress $\overline{\overline{\sigma}}$, for the sake of simplicity.

## FAILURE DESCRIPTION

Failure is generally associated to the notion of limit stress state. Such limit stress states are easily observable in homogeneous laboratory tests where some loading paths lead to stress states that cannot be exceeded. The drained triaxial compression on a dense granular assembly constitutes a classical example. Fig. 1a shows the simulated response with the discrete element model to a two-dimensionnal drained biaxial compression under a confining pressure $\sigma_2$ = 300 kPa. The axial stress $\sigma_1$ grows until reaching a maximum. This maximum constitutes a limit stress state. This point can be verified by switching the control and response parameters.

For the simulation we have just presented, the loading path was fixed by the loading parameter $d\sigma_2 = 0$ (defining a straight line in the stress plane) and the mechanical state of the granular assembly along

*Powders and Grains 2013*
AIP Conf. Proc. 1542, 585-588 (2013); doi: 10.1063/1.4811999

this path were controlled through the axial strain $\varepsilon_1$ by imposing an axial compression ($d\varepsilon_1 > 0$). The axial stress $\sigma_1$ was then a response parameter. We can renew this experiment by controlling $\sigma_1$ and imposing a stress increases ($d\sigma_1 > 0$), while $\varepsilon_1$ is the response parameter. The response to this loading program is superimposed to the previous one in Fig. 1a. Even though a constant increase of $\sigma_1$ is imposed, the limit stress state is not exceeded (actually it is here slightly exceeded due to inertial terms no more negligible at failure initiation). While it is approached, deformations increase strongly just as the strain rate and the kinetic energy as displayed in Fig. 1b. Thus, it is shown that the peak of $\sigma_1$ is a limit stress state and that failure (characterized by unlimited strains and a transition from a quasi-static response to a dynamic one [2]) effectively occurs when the load apply to the granular assembly exceed this limit state.

Limit stress states are classically described in soils by failure criterion of Mohr-Coulomb type. They can also be identified with the second-order work criterion stating that a limit state is reached if $W_2 \leq 0$ [3] (for the biaxial loading path $d\sigma_2 = 0$ is imposed and $W_2$ vanishes together with $d\sigma_1$). This last criterion is more general than the Mohr-Coulomb criterion since it offers the possibility to detect mechanical states strictly included within the Mohr-Coulomb criterion from which failure may develop. This has been deeply discussed in previous papers (for instance [3] & [4]).

Another important discussion in the description of failure is about the loss of homogeneity of the strain field and the development of strain localization patterns. For the drained compression previously discussed, the sample is initially in a dense state and as shown in Fig. 2, shear bands develop when the limit stress state is approached, indifferently for an axial stress or an axial strain control of the loading. Analytically, shear bands occurrence are detected with the Rice's criterion corresponding to the vanishing of the determinant of the acoustic tensor [5]. Consequently, the occurrence of failure characterized by strain localization pattern in shear bands will be detected along a given loading path when both criteria (second-order work for failure and Rice's criterion for shear band) are verified. Note that the effective occurrence of failure along the considered loading path depends on the mode of control of the loading (here for the drained compression an axial stress control is necessary).

However, limit states and failure can also be associated with homogeneous strain fields as shown in Fig. 3, where incremental deviatoric strain fields are displayed for a medium dense 2D particle assembly subjected to an undrained (isochoric) biaxial compression. For such a loading path the second-order

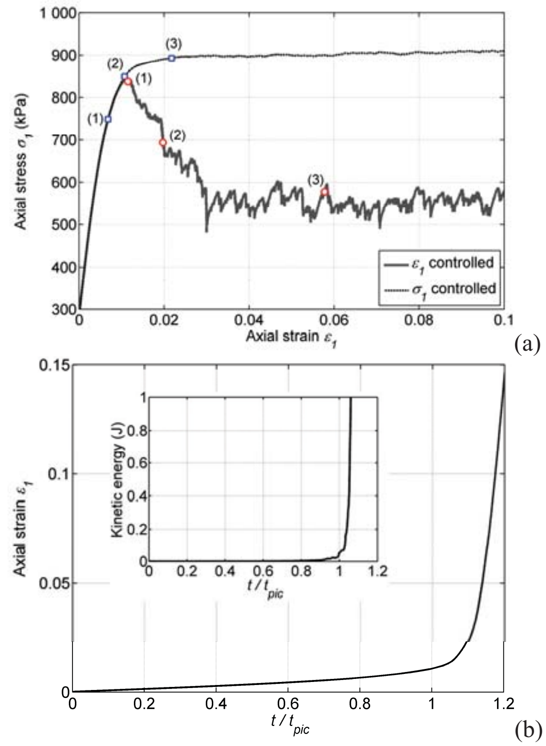


**FIGURE 1.** Responses of the discrete element model to a drained biaxial compression axially strain controlled (a), or stress controlled (a & b).

work writes $W_2 = V\, dq\, d\varepsilon_1$ and thus vanishes at the peak of the stress deviator $q$, corresponding to a limit state from which failure can effectively develops if the stress deviator $q$ is the control parameter. However strain field stays rather homogeneous after the peak of q, and we can imagine that Rice's criterion never holds along this loading path for this granular assembly.

## LOCAL SECOND-ORDER WORK

We consider a homogeneous volume $V_o$ of granular material comprised of $N$ grains. The shape of each grain '$p$' is arbitrary. The total number of contacts at time $t$ within the assembly is denoted $N_c$. The Lagrangian formulation given in Eq. (1) can be readily differentiated, then providing the following expression of the second-order work (see [6] for more details):

$$W_2 = \sum_{p,q} \delta f_i^c \ \delta l_i^c + \sum_{p \in V} \delta f_i^p \ \delta x_i^p \qquad (2)$$

where $\overline{l}^c$ is the branch vector relating the centres of contacting particles, $\overline{f}^c$ is the contact force between

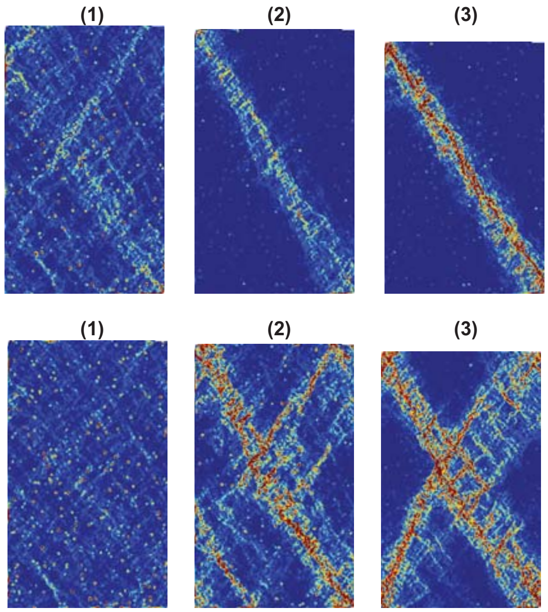


**FIGURE 2.** Incremental deviatoric strain fields computed at states numbered from 1 to 3 in Fig. 1, for the axially strain controlled loading (top) and stress controlled one (bottom); color represents the intensity of incremental deviatoric strain.

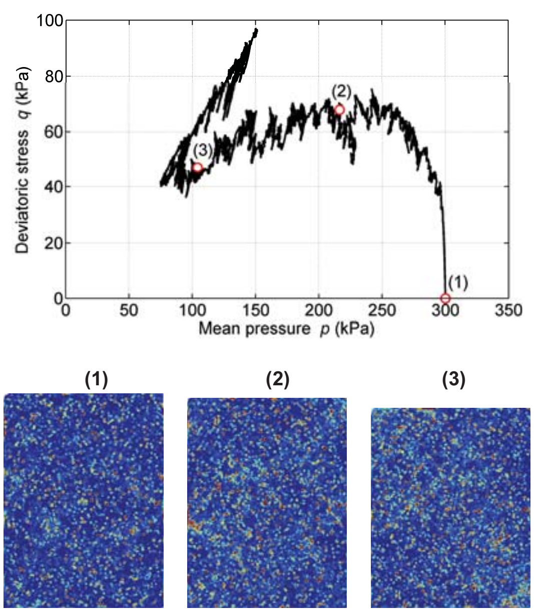


**FIGURE 3.** Stress response path for an undrained biaxial compression (top), and incremental deviatoric strain fields computed at states numbered from 1 to 3 (bottom)

contacting particles, and $\overline{f}^{p}$ denotes the resultant force applied to the particle 'p' of position $\overline{x}^{p}$. As specified in [6], the creation or the deletion of contacts is accounted for in this approach. The symbol $\sum_{p,q}$ denotes the summation over $p$ and $q$ varying over $[1, N]$ with $q \leq p$, and $c$ refers to the contacting pair $(p, q)$. When no contact exists between particles 'p' and 'q', $\overline{f}^{c}$ is set to zero.

It is worth noting that in the absence of incremental unbalanced force and in quasi-static regime, Eq. (2) simplifies into:

$$W_2 = \sum_{p,q} \delta f_i^c \ \delta l_i^c \qquad (3)$$

The validity of this relation has been numerically checked for 3D granular assemblies in axisymmetric conditions. After a first triaxial loading path under a confining pressure of 100 kPa, different strain probes were simulated from the same loading point (corresponding to a deviatoric ratio $\eta = 0.48$). The strain probes have the same amplitude ($10^{-4}$), and are characterized by different orientations $\alpha_\varepsilon$ in the Rendulic strain plane. The stress response is computed for each orientation $\alpha_\varepsilon$ and $W_2$ is deduced from Eq. (1). Likewise, the quantity $\sum_{p,q} \delta f_i^c \ \delta l_i^c$ can also be computed, which allows the validity of Eq. (3) to be assessed.

As seen in Fig. 4, an excellent agreement between both expressions of the second-order work with micro and macro variables is obtained. Equation (3) expresses the internal second-order work from micromechanical variables, namely the contact forces existing between contacting granules, and the branch vectors joining these granules. The attempt of such a formulation is to go down to the microscopic scale, to try to elucidate what are the basic microstructural origins giving rise to the vanishing of the internal second-order work, and therefore what are the microstructural contexts prone to instabilities.

To progress along this line, we plotted for the drained biaxial compressions presented in previous section the space distribution of inter-particle contacts $c^-$ where the local second-order work ($\delta f_i^c \ \delta l_i^c$) is negative in Fig. 5, for both axially strain and stress controlled loading paths. These distributions can be compared with the incremental deviatoric strain fields in Fig. 2. The patterns of these distributions are very similar, $c^-$ contacts are concentrated in the shear bands (as shown by the theory) where failure occurs and are sparsely distributed outside the shear band where the

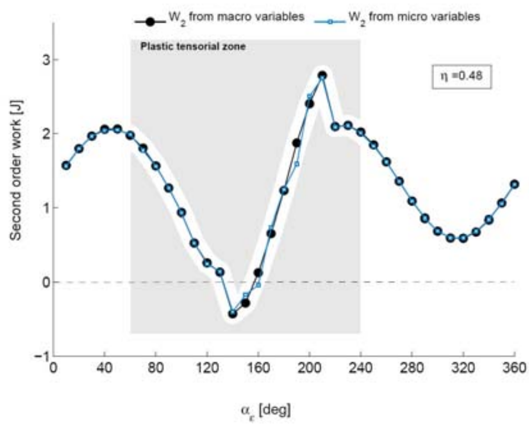


**FIGURE 4.** Macroscopic and microscopic expressions of the second-order work.

material is unloading. Inversely, $c^-$ contacts for the undrained loading path (presented in Fig. 3) stay homogeneously distributed, even after the point of potential failure occurrence ($q$ peak) as shown in Fig. 6.

## CONCLUSION

Effective failure can be described as the bifurcation of the response of the granular assembly from a quasi-static one, before failure initiation, to a dynamic one during failure development. Along a given loading path, the occurrence of this bifurcation (and thus of failure) depends on the mode of control of the granular assembly. Possible points of bifurcation are detected with the second-order work criterion. This later can be equivalently expressed in the framework of continuum mechanics or in the discrete micro-mechanics framework. Both expressions give at the scale of the granular assembly an identical information about the possibility of failure occurrence. Nevertheless the local expression of the second-order work criterion seems to give a richer information, since the spatial distribution of inter-particle contacts satisfying this criterion is apparently directly related to the mode of failure (localized or diffused). However, additional statistical analysis need to be performed to improve the understanding of potential links between failure mode and local second-order work.

## ACKNOWLEDGMENTS

The authors would like to express their sincere thanks to the French Research Network MeGe (Multiscale and multi-physics couplings in geo-environmental mechanics GDR CNRS 3176, 2008-2011) for having supported this work.

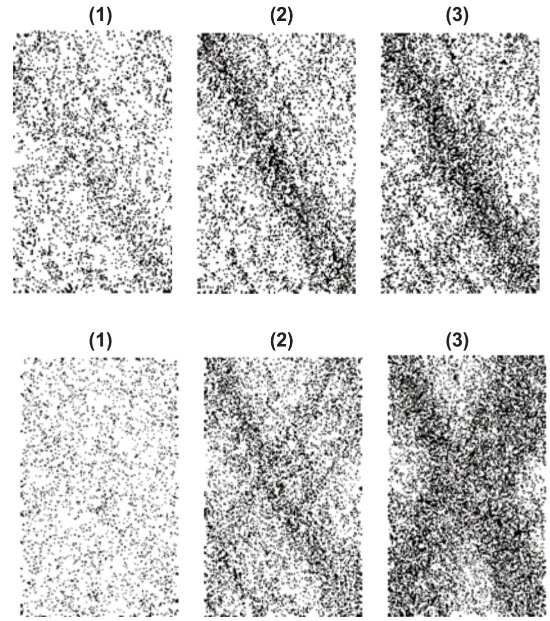


**FIGURE 5.** Fields of $c^-$ contacts for the drained biaxial compression axially strain controlled (top) and stress controlled (bottom).

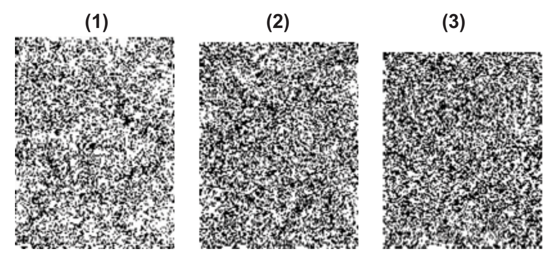


**FIGURE 6.** Fields of $c^-$ contacts for the undrained biaxial compression presented in Fig. 3.

## REFERENCES

1. Yade: Open source discrete element method. http://yade-dem.org.
2. F. Nicot, L. Sibille, F. Darve, *Int. Journal of Plasticity* **29**, 136-154 (2012).
3. F. Darve, G. Servant, F. Laouafa, H. Khoa, *Computer Meth. Appl. Mech. Eng.* **85** (27-29), 3057-3085 (2004).
4. F. Darve, L. Sibille, A. Daouadji, F. Nicot, *Comptes Rendus Mecanique* **335**, 496-515 (2007).
5. J.R. Rice, "The localization of plastic deformation", in *Theoretical and Applied Mechanics*, edited by W.T. Koiter, Delft: North-Holland Publishing Compagny, 1976, pp. 207-220.
6. F. Nicot, N. Hadda, F. Bourrier, L. Sibille, R. Wan, F. Darve, *Int. J. of Solids and Structures* **49** (10), 1252-1258 (2012).